P.S. Problem Solving

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

1. Finding a Limit Let R be the area of the region in the first quadrant bounded by the parabola $y = x^2$ and the line $y = cx$, $c > 0$. Let T be the area of the triangle AOB. Calculate the limit

$$\lim_{c \to 0^+} \frac{T}{R}.$$

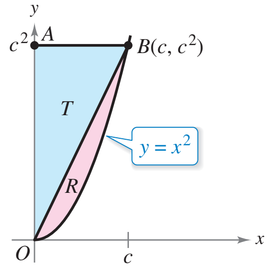


Figure for 1

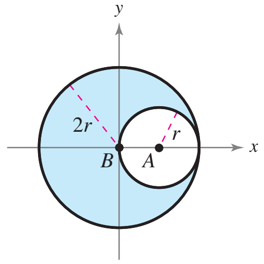


Figure for 2

2. Center of Mass of a Lamina Let L be the lamina of uniform density $\rho = 1$ obtained by removing circle A of radius r from circle B of radius $2r$ (see figure).

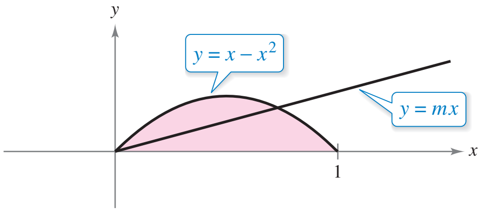
(a) Show that $M_x = 0$ for L.

(b) Show that M_y for L is equal to $(M_y \text{ for } B) - (M_y \text{ for } A)$.

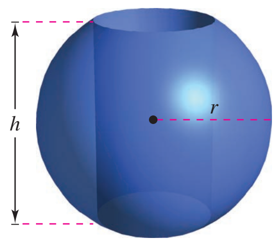
(c) Find M_y for B and M_y for A. Then use part (b) to compute M_y for L.

(d) What is the center of mass of L?

3. Dividing a Region Let R be the region bounded by the parabola $y = x - x^2$ and the x-axis. Find the equation of the line $y = mx$ that divides this region into two regions of equal area.



4. Volume A hole is cut through the center of a sphere of radius r (see figure). The height of the remaining spherical ring is h. Find the volume of the ring and show that it is independent of the radius of the sphere.



5. Surface Area Graph the curve

$$8y^2 = x^2(1 - x^2).$$

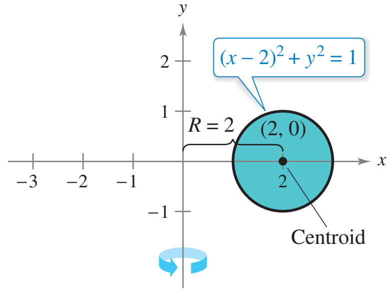
Use a computer algebra system to find the surface area of the solid of revolution obtained by revolving the curve about the y-axis.

6. Torus

(a) A torus is formed by revolving the region bounded by the circle

$$(x - 2)^2 + y^2 = 1$$

about the y-axis (see figure). Use the disk method to calculate the volume of the torus.



(b) Use the disk method to find the volume of the general torus when the circle has radius r and its center is R units from the axis of rotation.

7. Volume A rectangle R of length ℓ and width w is revolved about the line L (see figure). Find the volume of the resulting solid of revolution.

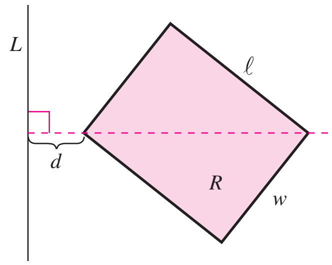


Figure for 7

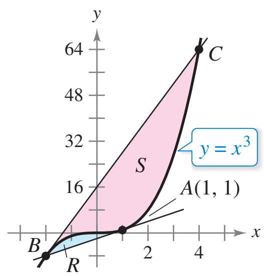


Figure for 8

8. Comparing Areas of Regions

(a) The tangent line to the curve $y = x^3$ at the point $A(1, 1)$ intersects the curve at another point B. Let R be the area of the region bounded by the curve and the tangent line. The tangent line at B intersects the curve at another point C (see figure). Let S be the area of the region bounded by the curve and this second tangent line. How are the areas R and S related?

(b) Repeat the construction in part (a) by selecting an arbitrary point A on the curve $y = x^3$. Show that the two areas R and S are always related in the same way.

9. Using Arc Length The graph of $y = f(x)$ passes through the origin. The arc length of the curve from $(0, 0)$ to $(x, f(x))$ is given by

$$s(x) = \int_0^x \sqrt{1 + e^t}\, dt.$$

Identify the function f.

10. Using a Function Let f be rectifiable on the interval $[a, b]$, and let

$$s(x) = \int_a^x \sqrt{1 + [f'(t)]^2}\, dt.$$

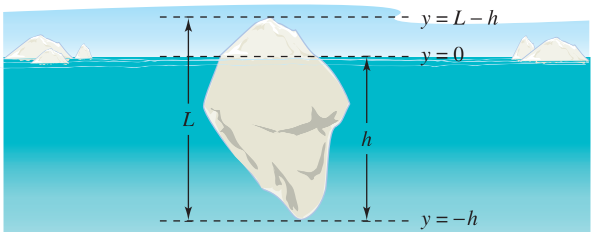
(a) Find $\dfrac{ds}{dx}$.

(b) Find ds and $(ds)^2$.

(c) Find $s(x)$ on $[1, 3]$ when $f(t) = t^{3/2}$.

(d) Use the function and interval in part (c) to calculate $s(2)$ and describe what it signifies.

11. Archimedes' Principle **Archimedes' Principle** states that the upward or buoyant force on an object within a fluid is equal to the weight of the fluid that the object displaces. For a partially submerged object, you can obtain information about the relative densities of the floating object and the fluid by observing how much of the object is above and below the surface. You can also determine the size of a floating object if you know the amount that is above the surface and the relative densities. You can see the top of a floating iceberg (see figure). The density of ocean water is 1.03×10^3 kilograms per cubic meter, and that of ice is 0.92×10^3 kilograms per cubic meter. What percent of the total iceberg is below the surface?



12. Finding a Centroid Sketch the region bounded on the left by $x = 1$, bounded above by $y = 1/x^3$, and bounded below by $y = -1/x^3$.

(a) Find the centroid of the region for $1 \le x \le 6$.

(b) Find the centroid of the region for $1 \le x \le b$.

(c) Where is the centroid as $b \to \infty$?

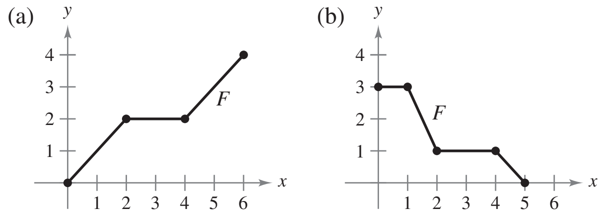
13. Finding a Centroid Sketch the region to the right of the y-axis, bounded above by $y = 1/x^4$, and bounded below by $y = -1/x^4$.

(a) Find the centroid of the region for $1 \le x \le 6$.

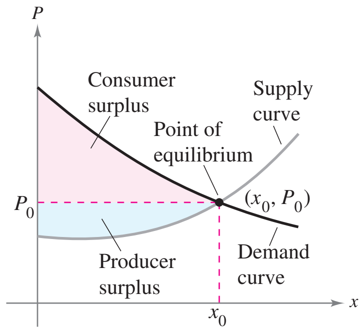
(b) Find the centroid of the region for $1 \le x \le b$.

(c) Where is the centroid as $b \to \infty$?

14. Work Find the work done by each force F.



Consumer and Producer Surplus **In Exercises 15 and 16, find the consumer surplus and producer surplus for the given demand $[p_1(x)]$ and supply $[p_2(x)]$ curves. The consumer surplus and producer surplus are represented by the areas shown in the figure.**



15. $p_1(x) = 50 - 0.5x, \quad p_2(x) = 0.125x$

16. $p_1(x) = 1000 - 0.4x^2, \quad p_2(x) = 42x$

17. Fluid Force A swimming pool is 20 feet wide, 40 feet long, 4 feet deep at one end, and 8 feet deep at the other end (see figures). The bottom is an inclined plane. Find the fluid force on each vertical wall.